

Hadronic Wave Functions in the Instanton Model

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Abstract

In this paper we wish to study hadronic wave functions using an instanton model for the QCD vacuum. The wave functions are defined in terms of gauge invariant Bethe Salpeter amplitudes which we have determined numerically using a Monte Carlo simulation of the instanton ensemble. We find that the pion and the proton, as well as the rho meson and the delta have very similar wavefunctions but observe a sizeable splitting between mesons or baryons with different spin. We compare our results with data obtained in lattice gauge simulations.

SUNY-NTG-94/5

*supported in part by the Alexander von Humboldt foundation.

1. Introduction

Hadronic correlation functions are an important tool in understanding the complicated nonperturbative structure of the QCD vacuum [1]. These correlation functions are defined as the vacuum expectation value $\Pi(x) = \langle T(j_h(x)j_h(0)) \rangle$ of the time ordered product of two currents with the quantum numbers of a given hadron h . At short distances, asymptotic freedom predicts that the correlation function is dominated by the propagation of free quarks. In this case, the correlator follows a simple power law behavior, $\Pi(x) \sim 1/x^6$ for mesons and $\Pi(x) \sim 1/x^9$ for baryons. Corrections to this behavior can be studied using the Operator Product Expansion.

In general the correlation function can be expressed in terms of the contributions of intermediate physical states. At large spacelike separations, the lowest resonance in a given channel dominates and the corresponding mass and coupling constant can be extracted from the exponential falloff of the correlator. Roughly speaking, the coupling constant measures the probability for all quarks inside the hadron to be at the same point. In order to obtain more detailed information about the structure of the hadron the concept of hadronic wavefunctions was introduced in [2]. In the case of the pion this quantity is defined as the following gauge invariant Bethe-Salpeter amplitude

$$\psi_\pi(y) = \int d^4x \langle 0 | \bar{d}(x) P e^{i \int_x^{x+y} A(x') dx'} \gamma_5 u(x+y) | \pi \rangle . \quad (1)$$

Here the two quarks are connected by a string of flux. One should note that in a relativistic theory the concept of a hadronic wavefunction is not uniquely defined. Other functions providing somewhat different information include the Bethe-Salpeter amplitudes in a fixed gauge or density-density correlation functions (inside a given hadron). In practice the

Bethe-Salpeter amplitude eq. (1) is extracted from the three point correlator

$$\Pi_\pi(x, y) = \langle 0 | T(\bar{d}(x) P e^{i \int_x^{x+y} A(x') dx'} \gamma_5 u(x+y) \bar{d}(0) \gamma_5 u(0)) | 0 \rangle \sim \psi(y) e^{-m_\pi x} \quad (2)$$

where x has to be a large spacelike separation in order to ensure that the correlation function is dominated by the ground state and y is the separation of the two quarks in the transverse direction.

The Bethe-Salpeter amplitude eq. (1) has been measured in a number of lattice gauge simulations, both at zero and at finite temperature [2, 3, 4, 5]. However, with the exception of one work in the context of the bag model [6] and the case of very large temperatures, where the wave function in the dimensionally reduced theory can be evaluated from an effective Schrödinger equation [7, 8], there have been no efforts to determine these wavefunctions in a theoretical model.

We would like to fill this gap by applying the Random Instanton Liquid Model [9, 10, 11] to the problem of calculating hadronic wavefunctions. The model is based on the assumption that the QCD vacuum is dominated by a particular type of gauge field configurations, small size instantons. The vacuum is then characterized by the density $n_0 = 1 \text{ fm}^4$ and the average size $\bar{\rho} = 1/3 \text{ fm}$ of the instantons. Essentially, these numbers were chosen to reproduce two global properties of the QCD vacuum, the value of the quark and gluon condensates. Recently, we have determined a large number of hadronic two point correlation functions using this model [11, 12]. The model not only successfully describes the masses and coupling constants of the lowest resonances in most channels, but also reproduces the results of a recent lattice calculation of two point correlation functions [13].

In the present work we want to extend this comparison to hadronic Bethe-Salpeter amplitudes. First of all, this will enable us to check whether the correlators calculated

in [11, 12] really correspond to the propagation of hadronic *bound* states. In addition to that, one can study whether the size and shape of the corresponding hadronic wave functions is similar to what was found in lattice simulations. In general, these are very non trivial questions. The Instanton Model takes into account only one very specific non perturbative effect among the many that determine the structure of QCD. Whether instanton induced effects really dominate the physics of a particular observable has to be checked in each individual case.

Another interesting question concerns the *differences* between the wavefunctions of hadronic states with different quantum numbers. In a potential model approach, these differences are due to the gluomagnetic forces that give rise to a spin-spin interaction between quarks. These forces are absent in our calculations. However, there also exists a spin-dependent instanton induced interaction between quarks, which qualitatively describes the mass splitting between different states. The question is whether it can also account for the differences in the size of the corresponding bound states.

Finally we would like to address a few questions related to the interpretation of lattice results on hadronic Bethe-Salpeter amplitudes. In particular we want to study the questions whether the observation of localized Bethe Salpeter amplitude uniquely reflects on the existence of a hadronic bound state and whether measurements of correlation functions at rather short distances on the order of one fermi (or even less) are sufficient to deduce properties of the ground state in a given channel.

2. Wave Functions in the Random Instanton Model

In the following we will study Bethe-Salpeter amplitudes for the pion, the rho meson,

the nucleon and the delta resonance. For this purpose we have considered correlation functions involving the following currents

$$j^\pi(x) = \bar{d}(x)\gamma_5 u(x), \quad (3)$$

$$j_\mu^\rho(x) = \bar{d}(x)\gamma_\mu u(x), \quad (4)$$

$$j_\alpha^p(x) = \epsilon^{abc}(u^a(x)C\gamma_5 d^b(x))u_\alpha^c(x), \quad (5)$$

$$j_{\alpha,\mu}^\Delta(x) = \epsilon^{abc}\left(2(u^a(x)C\gamma_\mu d^b(x))u_\alpha^c(x) + (u^a(x)C\gamma_\mu u^b(x))d_\alpha^c(x)\right). \quad (6)$$

Here q_α^a denotes a u or d -quark spinor with color index a and spinor index α and C is the charge conjugation matrix. The nucleon current given above is a linear combination of the standard Ioffe currents [12]. Our choice here is motivated by the fact that this current was used in the lattice simulations reported in [2]. We have done calculations using all six nucleon correlation functions introduced in [12] and will comment on the importance of the choice of current below.

We consider the case of exact isospin symmetry so we have simply chosen the flavor content of the mesonic currents to minimize the number of contractions contributing to the correlation function. In the case of the nucleon we have used a proton current and defined the corresponding wave function by taking the two u quarks at the same point and measuring the correlator as a function of the separation of the d quark. Similarly, for the delta resonance we have taken the Δ^+ state and again measured the Bethe Salpeter amplitude as a function of the distance between the two u quarks and the d quark.

The Bethe-Salpeter amplitude defined in eq. (1) involves a diagonal correlation function of two pseudoscalar currents. The same information, however, can also be obtained from non diagonal correlation functions. In the case of the pion, we use the pseudoscalar-

axialvector correlation function

$$\Pi_{\pi-a_1,\mu}(x,y) = \langle 0|T(\bar{d}(x)Pe^{i\int_x^{x+y}A(x')dx'}\gamma_5u(x+y)\bar{d}(0)\gamma_\mu\gamma_5u(0))|0\rangle, \quad (7)$$

while in the case of the rho meson, we have also determined the Bethe-Salpeter amplitude from the off diagonal vector-tensor correlation function

$$\Pi_{\rho-T,\mu}(x,y) = \langle 0|T(\bar{d}(x)Pe^{i\int_x^{x+y}A(x')dx'}\gamma_\nu u(x+y)\bar{d}(0)\sigma_{\mu\nu}u(0))|0\rangle. \quad (8)$$

Similarly, for the nucleon one can use non diagonal correlation functions between the two different Ioffe currents [12]. The advantage of using non diagonal correlators lies in the fact that these correlation functions receive no contribution from free quark propagation. For this reason the dominance of the ground state over excited states at large but finite separation is expected to be somewhat better for non diagonal correlators. In any case, comparing the Bethe Salpeter amplitudes determined from diagonal and non diagonal correlators provides a nice check on whether the measured wavefunctions really do reflect on the groundstate in the given channel.

In addition to the channels considered here it would also be interesting to study the wave functions of other mesons. Of particular interest in comparison with the rho meson is the a_1 meson. This channel, however, is difficult to analyze since the a_1 correlation function at large distances is dominated by the pion [1] and the off diagonal a_1 -tensor correlation function turns out to be very small. Furthermore, it would be nice to demonstrate that one does not observe localized Bethe Salpeter amplitudes in channels in which (at least inside the model) no bound states are expected to form. In the Instanton Model, one such channel would be the scalar isovector (δ -meson) channel. Again, the analysis turns out to be very difficult since the fact that the interaction is repulsive implies that the measured correlation function at large distances is very small so that the wave function cannot be reliably determined.

The instanton ensemble in QCD can be generally described by a partition function of the type

$$Z = \int \Pi_i [d\Omega_i \exp(-S_i)] \exp(-S_{int}) \Pi_{f=1, N_f} \det(i\hat{D} + im_f) \quad (9)$$

where $[d\Omega_i]$ denotes the integration over the collective coordinates (orientation, size, position) associated with the instantons, $S_i = 8\pi^2/g^2(\rho)$ is the action of an individual instanton and S_{int} is the instanton-instanton interaction. Since the evaluation of this partition function still constitutes a formidable problem we consider the even simpler Random Instanton Model. In this model we assume that except for the size, which we keep fixed, the distribution of the collective coordinates is completely random. In particular, adopting this assumption we neglect the effects of the fermion determinant $\det(i\hat{D} + im_f)$, which is similar to “quenched” lattice simulations we are going to compare with.

The Bethe-Salpeter amplitudes are then determined from the quark propagator in a given configuration. The pion correlation function eq. (2), for example, is given by

$$\Pi_\pi(x, y) = \langle \text{Tr} S^{ab}(x+y) (P e^{i \int_x^{x+y} A(x') dx'})^{bc} S^{ca}(-x) \rangle \quad (10)$$

where the trace is performed with respect to the Dirac indices. The calculation of the propagator is explained in detail in [11]. Here we only outline the general strategy. The quark propagator in the field of a single instanton can be written as the sum of a zero mode contribution and the propagator due to non zero modes. Both pieces are known analytically. In the many instanton configuration we treat the zero modes exactly by numerically inverting the Dirac operator in the space spanned by the zero modes of the individual instantons. The non zero modes, on the other hand, are taken into account only approximately, by including the first term in a multiple scattering expansion. The

resulting quark propagator is given by

$$S(x, y) = \sum_{IJ} \langle x | \phi_I \rangle \langle \phi_I | \frac{1}{i\gamma_\mu D_\mu + im} | \phi_J \rangle \langle \phi_J | y \rangle + S_{NZM}(x, y), \quad (11)$$

where $\phi_I(x)$ is the zero mode associated with the instanton I and

$$S_{NZM}(x, y) = S_0(x, y) + \sum_I (S_{NZM}^I(x, y) - S_0(x, y)) \quad (12)$$

is the propagator due to non zero modes. Here, S_0 denotes the free propagator and S_{NZM}^I is the non zero mode propagator in the field of an individual instanton. The non zero mode propagator also has to be corrected for the effects of current quark masses [11]. In this paper we take the light quark mass to be $m_{u,d} = 10$ MeV and the corresponding effects are small. The Schwinger factor $P \exp(i \int A_\mu dx_\mu)$ is calculated numerically for any given gauge field configuration by expanding the pathordered exponential as an infinite product

$$P \exp(i \int A_\mu dx_\mu) = \prod_i (1 + i A_\mu(x_i) dx_{i\mu}). \quad (13)$$

In practice this expression is approximated by a finite product where the stepsize dx_i is determined by the magnitude of the local gauge potential.

In general, the inclusion of the Schwinger factor eq. (13) is expected to give an important contribution to the measured wave functions, since it corresponds to an additional string type potential. The instanton liquid, however, does not produce confinement and no string potential is expected to appear. At large separations, the Schwinger factor decreases exponentially $P \exp(i \int_0^r A dx) \sim \exp(-mr)$ with m being the mass renormalization of a heavy quark in the instanton background [12]. Using the ‘sum ansatz’ for the gauge field of the multi-instanton configuration we find $m = 65$ MeV, in agreement with estimates made in [14]. Employing the more realistic ratio ansatz an even smaller

screening mass $m = 20$ MeV is found. In any case, the insertion of the path ordered exponential will only lead to minor effects on the wave functions we are going to discuss.

3. Results and comparison with Lattice Gauge Simulations

Before we come to the results obtained, let us discuss the Bethe Salpeter wave functions obtained by using simplified models for the propagator. Since in all numerical simulations to date the separation between the sources is rather modest (at most on the order of 1 fm) it is important to understand what the size of non asymptotic effects in the measured wave functions are.

In the simplest case we consider a free fermion propagator

$$S_0(x) = \frac{i}{2\pi^2} \frac{\gamma \cdot x}{x^4} \quad (14)$$

and take the path ordered exponential to be unity. Here all gamma matrices and coordinates are taken to be euclidean. Inserting this propagator in the pion correlator eq.(2) we get¹

$$\Pi_\pi(x, y) = \frac{N_c}{\pi^4} \frac{x^2 - \frac{y^2}{4}}{(x^2 + \frac{y^2}{4})^4} \simeq \frac{N_c}{\pi^4 x^6} \cdot (1 - \frac{5}{4} \frac{y^2}{x^2} + \dots). \quad (15)$$

At distances $\tau = \sqrt{x^2}$ on the order of one fermi, this corresponds to a 'wave function' with a mean square radius² $\sqrt{\langle r^2 \rangle} = 0.89$ fm, quite different from the asymptotic result. This effect becomes even more pronounced if one considers the propagation of a massive constituent quark

$$S(x) = i \frac{\gamma \cdot x}{\tau} D'(m_q, \tau) + m_q D(m_q, \tau) \quad (16)$$

¹Since we are discussing short range effects, we are using a symmetric configuration in which the quarks are located at transverse separations $-y/2, +y/2$. For asymptotic wave functions, this difference should not matter.

² The mean square radius is defined by $\langle r^2 \rangle = \int d^3r r^2 |\psi(r)|^2$ where the Bethe Salpeter wave function is normalized to one.

where $D(m, \tau) = m/(4\pi^2\tau)K_1(m\tau)$ is the coordinate space propagator of a massive scalar particle. Taking the quark mass to be 300 MeV, we find a wave function with a size of $\sqrt{\langle r^2 \rangle} = 0.79$ fm at a distance of $\tau = 1.0$ fm. Even at a distance of 1.5 fm, the wave function is still fairly well localized with a radius of $\sqrt{\langle r^2 \rangle} = 1.1$ fm. Note however, that so far the appearance of a localized wave function is a purely geometric effect and has nothing to do with the existence of bound states.

One should note that the fact that the Bethe Salpeter wavefunctions have not reached their asymptotic shape can not be detected from an analysis of the spacelike screening mass. The screening masses in this simple model are $m_{scr} = 2m_q$ for mesons and $m_{scr} = 3m_q$ for baryons. There are only small power corrections to this behavior that are due to the fact that the quarks can have some transverse momentum. We would also like to emphasize that considering the effects of finite quark masses on the Bethe Salpeter wave functions is very important at finite temperature where quarks develop thermal screening masses $m_{scr} = \pi T$. Around T_c this mass is on the order of 500 MeV. If the wave functions are determined at distances $x \sim 1/T$, which is typical for current lattice simulations, masses will have an important effect on the measured Bethe Salpeter wave functions.

After these remarks concerning non asymptotic effects we would like to study how instantons lead to the formation of bound states that manifest themselves in localized asymptotic Bethe Salpeter amplitudes. A simple one-instanton approximation that has been used [15] to analyze the qualitative effects of instantons on the *short-distance* behaviour of correlation functions is based on the following expression for the propagator in a dilute instanton gas

$$S(x) = S_0(x) + n_0 \int d^4z \frac{\phi_{0,z}(x)\phi_{0,z}^\dagger(0)}{-im^*}. \quad (17)$$

Here n_0 is the density of instantons, $\phi_{0,z}(x)$ is a zero mode localized at z and $m^* =$

$2\pi\rho_c\sqrt{n_0/6}$ is the effective quark mass in the instanton gas. Recall, that such propagator eq. (17) leads to an attractive pion correlation function and explains the failure of the operator product expansion in this channel. At distances greater than $\tau = 0.5$ fm, however, the full correlation function is strongly underestimated. In order to produce a real pion bound state one has to consider the corresponding four quark vertex and iterate this interaction in an RPA type approximation [16].

The zero modes are localized around the centers of the instantons with a characteristic size ρ . Evaluating the pion wave function eq. (1) using the propagator eq. (17) we find

$$\psi_\pi(y) = 1 - y^2/(2\rho)^2 + \dots \quad (18)$$

for small transverse separations y . The corresponding mean square radius is $\sqrt{\langle r^2 \rangle} = 1.41$ fm at $\tau = 1$ fm, but does not converge as $\tau \rightarrow \infty$. This means that individual instantons produce some interaction at short distances but the corresponding wave function has a long range tail and no true bound state exists.

A similar calculation can also be done for the nucleon. Using the proton current defined in eq. (5) and the model for the propagator introduced above, the nucleon correlation function simply factorizes into the product of a free quark propagator and a scalar diquark correlation function. The scalar diquark correlator is very similar to the pion one, in particular it is also attractive and the corresponding wave function as calculated from the propagator eq. (17) is the same. The isospin structure of the proton current is such that the d -quark which is used to define the Bethe Salpeter wave function is always in the scalar diquark. We then find that in this approximation the nucleon Bethe Salpeter wave function is identical to the one of the pion: $\psi_N(y) = \psi_\pi(y)$. This fact is essentially a consequence of the diquark structure of the nucleon current.

We have seen that such *single-instanton* approximation leads to a short range at-

Table 1: Rms radii extracted for the Bethe Salpeter wave functions measured at a longitudinal separation $\tau = 1.25$ fm. The various correlation functions are described in the text.

	$\pi(\text{P-P})$	$\pi(\text{P-AV})$	$\rho(\text{V-V})$	$\rho(\text{V-T})$
$\sqrt{\langle r^2 \rangle}$ [fm]	0.61	0.56	0.73	0.70
	$N(\eta_1 - \eta_1)$	$N(\eta_1 - \eta_2)$	Δ	
$\sqrt{\langle r^2 \rangle}$ [fm]	0.63	0.61	0.70	

traction which affects the wave functions, although no true bound state in the pion and nucleon channels appears. This is even more so for the rho meson and delta channels, in which individual instantons do not affect the correlator at all. In order to study the question whether an *ensemble* of instantons leads to the formation of hadronic bound states we have done numerical simulations in the Random Instanton Model. For these simulations we have studied an ensemble of 256 instantons in a box $6.7 \times 3.4^3 \text{ fm}^4$. In order to minimize finite size effects while still being able to study large separations, the correlation functions are measured along the long axis (the 'time' direction) of the box. For the measurement of wavefunctions, the available statistics limits us to the separations smaller than 1.5 fm.

The results of our calculations are shown in figures 1.-4. In fig. 1. we show the measured pion Bethe Salpeter wave function as a function of the longitudinal separation τ . The wave functions changes quite significantly between $\tau = 0.5$ fm and $\tau = 1$ fm, but remains roughly unchanged for $\tau > 1.0$ fm. We have extracted the corresponding mean square radii by assuming that the wave functions are exponential for transverse separations $y > 1.0$ fm. The pion rms radius measured this way is $\sqrt{\langle r_\pi^2 \rangle} = 0.61$ fm at $\tau = 1.25$ fm, significantly smaller than what we found for the dilute case considered above.

The fact that the Bethe Salpeter wave function appears to be dominated by the pion

is also illustrated by a comparison of the wave functions determined from the diagonal and off diagonal (pseudoscalar-axialvector) correlation functions (see fig. 2. and table 1.). The two wave functions are very similar with the off diagonal one being somewhat smaller. As explained in section 2 this might be due to the fact that there is still some contribution from the continuum left in the diagonal correlator while it is absent in the off diagonal one.

In fig. 2. we also show a similar comparison for the rho meson wave function determined from the vector and vector-tensor correlators as well as the nucleon wave function measured from the diagonal and the off diagonal correlator of the first and second Ioffe current. In both cases the agreement is excellent and we conclude that the wave functions are likely to be resonance dominated. This can also be seen from the fact that even in the rho meson channel, in which there is no first order instanton induced interaction, the wave function is smaller than what would be expected from the propagation of massive constituent quarks.

A comparison of the wave functions obtained in different channels is shown in fig. 3. We observe that the pion and the proton as well as the rho meson and the delta resonance have very similar wave functions. The pion and the proton, however, are significantly smaller than the rho meson and the delta resonance. We have already argued that the scalar diquark content of the nucleon and the fact that instantons produce substantial attraction in the scalar diquark channel suggest that the nucleon and pion wave functions are very similar. Analogously, one can think of the delta as consisting of a quark and a vector diquark. In this case, however, the above argument does not really apply since the d quark is not necessarily sitting in the diquark and there is no interaction that would enhance the importance of the quark-diquark part of the correlation function.

In fig. 4. we compare our results for the pion and the proton with the lattice result reported in [2, 3]. These authors have determined the Bethe Salpeter wave functions for the pion, the rho meson and the nucleon in a quenched lattice simulation. In addition to that, they have also measured wave functions after ‘cooling’ their configurations. This procedure relaxes any given gauge field configuration to the closest classical solution [17]. It removes essentially all gluon fluctuations except for those stabilized by topology. ‘Cooling’ therefore displays the contribution of instantons to the measured wave functions. Comparing the different curves shown in fig. 4. we observe that the full lattice wave functions are more compact than the instanton results while the cooled wave functions are even larger. Qualitatively it is clear why the wave functions in the instanton model extend farther out: the instanton model misses at least two effects, the Coulomb force at short distances and the string potential at large distances. Both help to localize the wave functions, although none of them seems to be of crucial importance.

This can be seen in more detail by considering the shape of the wave functions. At short distances our wave functions are quadratic in the separation, while the lattice results appear to be linear. This effect should be due to the perturbative Coulomb and spin-spin forces, since the linear behavior disappears when the wave functions are cooled. This can be checked further by applying only a few cooling sweeps (compared to the 25-50 sweeps used to extract the instanton liquid): the perturbative “quantum noise” disappears already at this level.

Finally, although the shapes are similar, the absolute size of the wave functions extracted from our model and from cooled lattice configurations do not agree. A possible reason for this behavior is the fact that there are some ambiguities related to the redefinition of the lattice scale after cooling. Even before cooling, the lattice scale depends on

the particular observable that is used to fix the units [2].

4. Conclusions

We have measured Bethe Salpeter wave functions for the pion, the proton, the rho meson and the delta resonance in the Random Instanton Liquid Model.

In all the channels considered here there is evidence that this Model leads to the formation of localized wave functions. This is based on the fact that the wave functions become stable as the longitudinal separation is increased and the observation that the wave functions measured using different sources are very similar to each other.

Furthermore, the size of these bound states are quite similar to those observed experimentally and measured on the lattice. In fact, the model reproduces most of the qualitative features of the wave functions that are observed in lattice simulations. In particular, we find that the wave functions of the pion and the proton are very similar whereas the rho meson has a significantly larger size.

The shape of the wave functions measured on the lattice are somewhat different, though. This is an important observation since the model does not include some of the physics both at very short and at very large distances, namely the perturbative and the confining forces. The size of the observed deviations is therefore a measure for the importance of these effects in the formation of hadronic bound states. After cooling the qualitative behavior of the lattice results agrees with what is observed in the Instanton Model. However, the mean square radii determined in the Instanton Model are somewhat smaller as compared to the cooled lattice reported in [3].

In conclusion we have shown that the Random Instanton Liquid Model can account

for the qualitative behavior of hadronic Bethe Salpeter wave functions. It will therefore be of great interest to extend the calculations presented in this work to include the effects of finite temperature.

5. Acknowledgements

The reported work was partially supported by the US DOE grant DE-FG-88ER40388. We acknowledge the NERSC at Lawrence Livermore where most of the computations presented in this paper were performed. We would like to thank Jac Verbaarschot for his collaboration on the Instanton Model, and very helpful discussions with John Negele on lattice results.

References

- [1] E. V. Shuryak, Rev. Mod. Phys. **65**, 1 (1993)
- [2] M.-C. Chu, M. Lissia, J. W. Negele, Nucl. Phys. **B360**, 31 (1991)
- [3] J. W. Negele, Nucl. Phys. **A553**, 47c (1993)
- [4] C. Bernard, T. A. DeGrand, C. DeTar, S. Gottlieb, A. Krasnitz, M. C. Ogilvie, R. L. Sugar, D. Toussaint, Phys. Rev. Lett. **68**, 2125 (1992)
- [5] S. Schramm, M.-C. Chu, Phys. Rev. **D48**, 2279
- [6] M. Lissia, M. C. Chu, J. W. Negele, J. M. Grandy, preprint, MIT-CTP#1922
- [7] V. Koch, E. V. Shuryak, G. E. Brown, A. D. Jackson, Phys. Rev. **D 46**, 3169 (1992)
V. Koch, preprint, SUNY-NTG-94-3

- [8] T. H. Hanson, I. Zahed, Nucl. Phys. **B374**, 227 (1992)
- [9] E. V. Shuryak, Nucl. Phys. **B203**, 116, 140, 237 (1982)
- [10] E. V. Shuryak, J. J. M. Verbaarschot, Nucl. Phys. **B** (1994), in press
- [11] E. V. Shuryak, J. J. M. Verbaarschot, Nucl. Phys. **B** (1994), in press
- [12] T. Schäfer, E. V. Shuryak, J. J. M. Verbaarschot, Nucl. Phys. **B** (1994), in press
- [13] M. C. Chu, J. M. Grandy, S. Huang, and J. W. Negele, Phys. Rev. Lett. **70**, 255 (1993); preprint, MIT-CTP#2148, hep-lat/9306002
- [14] D. I. Diakonov, V. Yu. Petrov, P. V. Pobylitsa, Phys. Lett. **B226**, 372 (1989)
- [15] E. V. Shuryak, Nucl. Phys. **B214**, 237 (1983)
- [16] D. I. Diakonov, V. Yu. Petrov, Nucl. Phys. **B245**, 259 (1984)
- [17] M. C. Chu, J. M. Grandy, S. Huang, and J. W. Negele, preprint, MIT-CTP#2269, hep-lat/9312071

Figure 1: Pion Bethe Salpeter wave function measured at different separations $\tau = 0.5, 0.75, 1.0$ and 1.25 fm. All wave functions are normalized to one at the origin. The lowest curve corresponds to $\tau = 0.5$ fm. The solid curves show a parametrization used to extract the means square radii.

Figure 2: Bethe Salpeter wave functions for the pion, the rho meson and the nucleon as determined from diagonal and off diagonal correlation functions at $\tau = 1.25$ fm. The different correlators used are explained in the text. All wave functions are normalized to one at the origin.

Figure 3: Bethe Salpeter amplitudes for the pion, the rho meson, the nucleon and the delta measured in the random instanton model. The wave functions are determined from diagonal correlation functions at $\tau = 1.25$ fm. They are normalized to one at the origin. The solid curves show a parametrization of the data used to extract the corresponding means square radii.

Figure 4: Comparison of pion and proton Bethe Salpeter amplitudes calculated in the random instanton model with the corresponding results of lattice calculations reported in [2,3]. The lowest curve is the full lattice result, the curve in the middle shows the result in the instanton model and the upper curve corresponds to the lattice result after cooling.

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SUNY-NTG-94/5

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1. Introduction

Hadronic correlation functions are an important tool in understanding the complicated nonperturbative structure of the QCD vacuum [1]. These correlation functions are defined as the vacuum expectation value $\Pi(x) = \langle T(j_h(x)j_h(0)) \rangle$ of the time ordered product of two currents with the quantum numbers of a given hadron h . At short distances, asymptotic freedom predicts that the correlation function is dominated by the propagation of free quarks. In this case, the correlator follows a simple power law behavior, $\Pi(x) \sim 1/x^6$ for mesons and $\Pi(x) \sim 1/x^9$ for baryons. Corrections to this behavior can be studied using the Operator Product Expansion.

In general the correlation function can be expressed in terms of the contributions of intermediate physical states. At large spacelike separations, the lowest resonance in a given channel dominates and the corresponding mass and coupling constant can be extracted from the exponential falloff of the correlator. Roughly speaking, the coupling constant measures the probability for all quarks inside the hadron to be at the same point. In order to obtain more detailed information about the structure of the hadron the concept of hadronic wavefunctions was introduced in [2]. In the case of the pion this quantity is defined as the following gauge invariant Bethe-Salpeter amplitude

$$\psi_\pi(y) = \int d^4x \langle 0 | \bar{d}(x) P e^{i \int_x^{x+y} A(x') dx'} \gamma_5 u(x+y) | \pi \rangle . \quad (1)$$

Here the two quarks are connected by a string of flux. One should note that in a relativistic theory the concept of a hadronic wavefunction is not uniquely defined. Other functions providing somewhat different information include the Bethe-Salpeter amplitudes in a fixed gauge or density-density correlation functions (inside a given hadron). In practice the

Bethe-Salpeter amplitude eq. (1) is extracted from the three point correlator

$$\Pi_\pi(x, y) = \langle 0 | T(\bar{d}(x) P e^{i \int_x^{x+y} A(x') dx'} \gamma_5 u(x+y) \bar{d}(0) \gamma_5 u(0)) | 0 \rangle \sim \psi(y) e^{-m_\pi x} \quad (2)$$

where x has to be a large spacelike separation in order to ensure that the correlation function is dominated by the ground state and y is the separation of the two quarks in the transverse direction.

The Bethe-Salpeter amplitude eq. (1) has been measured in a number of lattice gauge simulations, both at zero and at finite temperature [2, 3, 4, 5]. However, with the exception of one work in the context of the bag model [6] and the case of very large temperatures, where the wave function in the dimensionally reduced theory can be evaluated from an effective Schrödinger equation [7, 8], there have been no efforts to determine these wavefunctions in a theoretical model.

We would like to fill this gap by applying the Random Instanton Liquid Model [9, 10, 11] to the problem of calculating hadronic wavefunctions. The model is based on the assumption that the QCD vacuum is dominated by a particular type of gauge field configurations, small size instantons. The vacuum is then characterized by the density $n_0 = 1 \text{ fm}^4$ and the average size $\bar{\rho} = 1/3 \text{ fm}$ of the instantons. Essentially, these numbers were chosen to reproduce two global properties of the QCD vacuum, the value of the quark and gluon condensates. Recently, we have determined a large number of hadronic two point correlation functions using this model [11, 12]. The model not only successfully describes the masses and coupling constants of the lowest resonances in most channels, but also reproduces the results of a recent lattice calculation of two point correlation functions [13].

In the present work we want to extend this comparison to hadronic Bethe-Salpeter amplitudes. First of all, this will enable us to check whether the correlators calculated

in [11, 12] really correspond to the propagation of hadronic *bound* states. In addition to that, one can study whether the size and shape of the corresponding hadronic wave functions is similar to what was found in lattice simulations. In general, these are very non trivial questions. The Instanton Model takes into account only one very specific non perturbative effect among the many that determine the structure of QCD. Whether instanton induced effects really dominate the physics of a particular observable has to be checked in each individual case.

Another interesting question concerns the *differences* between the wavefunctions of hadronic states with different quantum numbers. In a potential model approach, these differences are due to the gluomagnetic forces that give rise to a spin-spin interaction between quarks. These forces are absent in our calculations. However, there also exists a spin-dependent instanton induced interaction between quarks, which qualitatively describes the mass splitting between different states. The question is whether it can also account for the differences in the size of the corresponding bound states.

Finally we would like to address a few questions related to the interpretation of lattice results on hadronic Bethe-Salpeter amplitudes. In particular we want to study the questions whether the observation of localized Bethe Salpeter amplitude uniquely reflects on the existence of a hadronic bound state and whether measurements of correlation functions at rather short distances on the order of one fermi (or even less) are sufficient to deduce properties of the ground state in a given channel.

2. Wave Functions in the Random Instanton Model

In the following we will study Bethe-Salpeter amplitudes for the pion, the rho meson,

the nucleon and the delta resonance. For this purpose we have considered correlation functions involving the following currents

$$j^\pi(x) = \bar{d}(x)\gamma_5 u(x), \quad (3)$$

$$j_\mu^\rho(x) = \bar{d}(x)\gamma_\mu u(x), \quad (4)$$

$$j_\alpha^p(x) = \epsilon^{abc}(u^a(x)C\gamma_5 d^b(x))u_\alpha^c(x), \quad (5)$$

$$j_{\alpha,\mu}^\Delta(x) = \epsilon^{abc}\left(2(u^a(x)C\gamma_\mu d^b(x))u_\alpha^c(x) + (u^a(x)C\gamma_\mu u^b(x))d_\alpha^c(x)\right). \quad (6)$$

Here q_α^a denotes a u or d -quark spinor with color index a and spinor index α and C is the charge conjugation matrix. The nucleon current given above is a linear combination of the standard Ioffe currents [12]. Our choice here is motivated by the fact that this current was used in the lattice simulations reported in [2]. We have done calculations using all six nucleon correlation functions introduced in [12] and will comment on the importance of the choice of current below.

We consider the case of exact isospin symmetry so we have simply chosen the flavor content of the mesonic currents to minimize the number of contractions contributing to the correlation function. In the case of the nucleon we have used a proton current and defined the corresponding wave function by taking the two u quarks at the same point and measuring the correlator as a function of the separation of the d quark. Similarly, for the delta resonance we have taken the Δ^+ state and again measured the Bethe Salpeter amplitude as a function of the distance between the two u quarks and the d quark.

The Bethe-Salpeter amplitude defined in eq. (1) involves a diagonal correlation function of two pseudoscalar currents. The same information, however, can also be obtained from non diagonal correlation functions. In the case of the pion, we use the pseudoscalar-

axialvector correlation function

$$\Pi_{\pi-a_1,\mu}(x,y) = \langle 0|T(\bar{d}(x)Pe^{i\int_x^{x+y}A(x')dx'}\gamma_5 u(x+y)\bar{d}(0)\gamma_\mu\gamma_5 u(0))|0\rangle, \quad (7)$$

while in the case of the rho meson, we have also determined the Bethe-Salpeter amplitude from the off diagonal vector-tensor correlation function

$$\Pi_{\rho-T,\mu}(x,y) = \langle 0|T(\bar{d}(x)Pe^{i\int_x^{x+y}A(x')dx'}\gamma_\nu u(x+y)\bar{d}(0)\sigma_{\mu\nu}u(0))|0\rangle. \quad (8)$$

Similarly, for the nucleon one can use non diagonal correlation functions between the two different Ioffe currents [12]. The advantage of using non diagonal correlators lies in the fact that these correlation functions receive no contribution from free quark propagation. For this reason the dominance of the ground state over excited states at large but finite separation is expected to be somewhat better for non diagonal correlators. In any case, comparing the Bethe Salpeter amplitudes determined from diagonal and non diagonal correlators provides a nice check on whether the measured wavefunctions really do reflect on the groundstate in the given channel.

In addition to the channels considered here it would also be interesting to study the wave functions of other mesons. Of particular interest in comparison with the rho meson is the a_1 meson. This channel, however, is difficult to analyze since the a_1 correlation function at large distances is dominated by the pion [1] and the off diagonal a_1 -tensor correlation function turns out to be very small. Furthermore, it would be nice to demonstrate that one does not observe localized Bethe Salpeter amplitudes in channels in which (at least inside the model) no bound states are expected to form. In the Instanton Model, one such channel would be the scalar isovector (δ -meson) channel. Again, the analysis turns out to be very difficult since the fact that the interaction is repulsive implies that the measured correlation function at large distances is very small so that the wave function cannot be reliably determined.

The instanton ensemble in QCD can be generally described by a partition function of the type

$$Z = \int \Pi_i [d\Omega_i \exp(-S_i)] \exp(-S_{int}) \Pi_{f=1, N_f} \det(i\hat{D} + im_f) \quad (9)$$

where $[d\Omega_i]$ denotes the integration over the collective coordinates (orientation, size, position) associated with the instantons, $S_i = 8\pi^2/g^2(\rho)$ is the action of an individual instanton and S_{int} is the instanton-instanton interaction. Since the evaluation of this partition function still constitutes a formidable problem we consider the even simpler Random Instanton Model. In this model we assume that except for the size, which we keep fixed, the distribution of the collective coordinates is completely random. In particular, adopting this assumption we neglect the effects of the fermion determinant $\det(i\hat{D} + im_f)$, which is similar to “quenched” lattice simulations we are going to compare with.

The Bethe-Salpeter amplitudes are then determined from the quark propagator in a given configuration. The pion correlation function eq. (2), for example, is given by

$$\Pi_\pi(x, y) = \langle \text{Tr} S^{ab}(x + y) (P e^{i \int_x^{x+y} A(x') dx'})^{bc} S^{ca}(-x) \rangle \quad (10)$$

where the trace is performed with respect to the Dirac indices. The calculation of the propagator is explained in detail in [11]. Here we only outline the general strategy. The quark propagator in the field of a single instanton can be written as the sum of a zero mode contribution and the propagator due to non zero modes. Both pieces are known analytically. In the many instanton configuration we treat the zero modes exactly by numerically inverting the Dirac operator in the space spanned by the zero modes of the individual instantons. The non zero modes, on the other hand, are taken into account only approximately, by including the first term in a multiple scattering expansion. The

resulting quark propagator is given by

$$S(x, y) = \sum_{IJ} \langle x | \phi_I \rangle \langle \phi_I | \frac{1}{i\gamma_\mu D_\mu + im} | \phi_J \rangle \langle \phi_J | y \rangle + S_{NZM}(x, y), \quad (11)$$

where $\phi_I(x)$ is the zero mode associated with the instanton I and

$$S_{NZM}(x, y) = S_0(x, y) + \sum_I (S_{NZM}^I(x, y) - S_0(x, y)) \quad (12)$$

is the propagator due to non zero modes. Here, S_0 denotes the free propagator and S_{NZM}^I is the non zero mode propagator in the field of an individual instanton. The non zero mode propagator also has to be corrected for the effects of current quark masses [11]. In this paper we take the light quark mass to be $m_{u,d} = 10$ MeV and the corresponding effects are small. The Schwinger factor $P \exp(i \int A_\mu dx_\mu)$ is calculated numerically for any given gauge field configuration by expanding the pathordered exponential as an infinite product

$$P \exp(i \int A_\mu dx_\mu) = \prod_i (1 + i A_\mu(x_i) dx_{i\mu}). \quad (13)$$

In practice this expression is approximated by a finite product where the stepsize dx_i is determined by the magnitude of the local gauge potential.

In general, the inclusion of the Schwinger factor eq. (13) is expected to give an important contribution to the measured wave functions, since it corresponds to an additional string type potential. The instanton liquid, however, does not produce confinement and no string potential is expected to appear. At large separations, the Schwinger factor decreases exponentially $P \exp(i \int_0^r A dx) \sim \exp(-mr)$ with m being the mass renormalization of a heavy quark in the instanton background [12]. Using the ‘sum ansatz’ for the gauge field of the multi-instanton configuration we find $m = 65$ MeV, in agreement with estimates made in [14]. Employing the more realistic ratio ansatz an even smaller

screening mass $m = 20$ MeV is found. In any case, the insertion of the path ordered exponential will only lead to minor effects on the wave functions we are going to discuss.

3. Results and comparison with Lattice Gauge Simulations

Before we come to the results obtained, let us discuss the Bethe Salpeter wave functions obtained by using simplified models for the propagator. Since in all numerical simulations to date the separation between the sources is rather modest (at most on the order of 1 fm) it is important to understand what the size of non asymptotic effects in the measured wave functions are.

In the simplest case we consider a free fermion propagator

$$S_0(x) = \frac{i}{2\pi^2} \frac{\gamma \cdot x}{x^4} \quad (14)$$

and take the path ordered exponential to be unity. Here all gamma matrices and coordinates are taken to be euclidean. Inserting this propagator in the pion correlator eq.(2) we get¹

$$\Pi_\pi(x, y) = \frac{N_c}{\pi^4} \frac{x^2 - \frac{y^2}{4}}{(x^2 + \frac{y^2}{4})^4} \simeq \frac{N_c}{\pi^4 x^6} \cdot (1 - \frac{5}{4} \frac{y^2}{x^2} + \dots). \quad (15)$$

At distances $\tau = \sqrt{x^2}$ on the order of one fermi, this corresponds to a 'wave function' with a mean square radius² $\sqrt{\langle r^2 \rangle} = 0.89$ fm, quite different from the asymptotic result. This effect becomes even more pronounced if one considers the propagation of a massive constituent quark

$$S(x) = i \frac{\gamma \cdot x}{\tau} D'(m_q, \tau) + m_q D(m_q, \tau) \quad (16)$$

¹Since we are discussing short range effects, we are using a symmetric configuration in which the quarks are located at transverse separations $-y/2, +y/2$. For asymptotic wave functions, this difference should not matter.

²The mean square radius is defined by $\langle r^2 \rangle = \int d^3r r^2 |\psi(r)|^2$ where the Bethe Salpeter wave function is normalized to one.

where $D(m, \tau) = m/(4\pi^2\tau)K_1(m\tau)$ is the coordinate space propagator of a massive scalar particle. Taking the quark mass to be 300 MeV, we find a wave function with a size of $\sqrt{\langle r^2 \rangle} = 0.79$ fm at a distance of $\tau = 1.0$ fm. Even at a distance of 1.5 fm, the wave function is still fairly well localized with a radius of $\sqrt{\langle r^2 \rangle} = 1.1$ fm. Note however, that so far the appearance of a localized wave function is a purely geometric effect and has nothing to do with the existence of bound states.

One should note that the fact that the Bethe Salpeter wavefunctions have not reached their asymptotic shape can not be detected from an analysis of the spacelike screening mass. The screening masses in this simple model are $m_{scr} = 2m_q$ for mesons and $m_{scr} = 3m_q$ for baryons. There are only small power corrections to this behavior that are due to the fact that the quarks can have some transverse momentum. We would also like to emphasize that considering the effects of finite quark masses on the Bethe Salpeter wave functions is very important at finite temperature where quarks develop thermal screening masses $m_{scr} = \pi T$. Around T_c this mass is on the order of 500 MeV. If the wave functions are determined at distances $x \sim 1/T$, which is typical for current lattice simulations, masses will have an important effect on the measured Bethe Salpeter wave functions.

After these remarks concerning non asymptotic effects we would like to study how instantons lead to the formation of bound states that manifest themselves in localized asymptotic Bethe Salpeter amplitudes. A simple one-instanton approximation that has been used [15] to analyze the qualitative effects of instantons on the *short-distance* behaviour of correlation functions is based on the following expression for the propagator in a dilute instanton gas

$$S(x) = S_0(x) + n_0 \int d^4z \frac{\phi_{0,z}(x)\phi_{0,z}^\dagger(0)}{-im^*}. \quad (17)$$

Here n_0 is the density of instantons, $\phi_{0,z}(x)$ is a zero mode localized at z and $m^* =$

$2\pi\rho_c\sqrt{n_0/6}$ is the effective quark mass in the instanton gas. Recall, that such propagator eq. (17) leads to an attractive pion correlation function and explains the failure of the operator product expansion in this channel. At distances greater than $\tau = 0.5$ fm, however, the full correlation function is strongly underestimated. In order to produce a real pion bound state one has to consider the corresponding four quark vertex and iterate this interaction in an RPA type approximation [16].

The zero modes are localized around the centers of the instantons with a characteristic size ρ . Evaluating the pion wave function eq. (1) using the propagator eq. (17) we find

$$\psi_\pi(y) = 1 - y^2/(2\rho)^2 + \dots \quad (18)$$

for small transverse separations y . The corresponding mean square radius is $\sqrt{\langle r^2 \rangle} = 1.41$ fm at $\tau = 1$ fm, but does not converge as $\tau \rightarrow \infty$. This means that individual instantons produce some interaction at short distances but the corresponding wave function has a long range tail and no true bound state exists.

A similar calculation can also be done for the nucleon. Using the proton current defined in eq. (5) and the model for the propagator introduced above, the nucleon correlation function simply factorizes into the product of a free quark propagator and a scalar diquark correlation function. The scalar diquark correlator is very similar to the pion one, in particular it is also attractive and the corresponding wave function as calculated from the propagator eq. (17) is the same. The isospin structure of the proton current is such that the d -quark which is used to define the Bethe Salpeter wave function is always in the scalar diquark. We then find that in this approximation the nucleon Bethe Salpeter wave function is identical to the one of the pion: $\psi_N(y) = \psi_\pi(y)$. This fact is essentially a consequence of the diquark structure of the nucleon current.

We have seen that such *single-instanton* approximation leads to a short range at-

Table 1: Rms radii extracted for the Bethe Salpeter wave functions measured at a longitudinal separation $\tau = 1.25$ fm. The various correlation functions are described in the text.

	$\pi(\text{P-P})$	$\pi(\text{P-AV})$	$\rho(\text{V-V})$	$\rho(\text{V-T})$
$\sqrt{\langle r^2 \rangle}$ [fm]	0.61	0.56	0.73	0.70
	$N(\eta_1 - \eta_1)$	$N(\eta_1 - \eta_2)$	Δ	
$\sqrt{\langle r^2 \rangle}$ [fm]	0.63	0.61	0.70	

traction which affects the wave functions, although no true bound state in the pion and nucleon channels appears. This is even more so for the rho meson and delta channels, in which individual instantons do not affect the correlator at all. In order to study the question whether an *ensemble* of instantons leads to the formation of hadronic bound states we have done numerical simulations in the Random Instanton Model. For these simulations we have studied an ensemble of 256 instantons in a box $6.7 \times 3.4^3 \text{ fm}^4$. In order to minimize finite size effects while still being able to study large separations, the correlation functions are measured along the long axis (the 'time' direction) of the box. For the measurement of wavefunctions, the available statistics limits us to the separations smaller than 1.5 fm.

The results of our calculations are shown in figures 1.-4. In fig. 1. we show the measured pion Bethe Salpeter wave function as a function of the longitudinal separation τ . The wave functions changes quite significantly between $\tau = 0.5$ fm and $\tau = 1$ fm, but remains roughly unchanged for $\tau > 1.0$ fm. We have extracted the corresponding mean square radii by assuming that the wave functions are exponential for transverse separations $y > 1.0$ fm. The pion rms radius measured this way is $\sqrt{\langle r_\pi^2 \rangle} = 0.61$ fm at $\tau = 1.25$ fm, significantly smaller than what we found for the dilute case considered above.

The fact that the Bethe Salpeter wave function appears to be dominated by the pion

is also illustrated by a comparison of the wave functions determined from the diagonal and off diagonal (pseudoscalar-axialvector) correlation functions (see fig. 2. and table 1.). The two wave functions are very similar with the off diagonal one being somewhat smaller. As explained in section 2 this might be due to the fact that there is still some contribution from the continuum left in the diagonal correlator while it is absent in the off diagonal one.

In fig. 2. we also show a similar comparison for the rho meson wave function determined from the vector and vector-tensor correlators as well as the nucleon wave function measured from the diagonal and the off diagonal correlator of the first and second Ioffe current. In both cases the agreement is excellent and we conclude that the wave functions are likely to be resonance dominated. This can also be seen from the fact that even in the rho meson channel, in which there is no first order instanton induced interaction, the wave function is smaller than what would be expected from the propagation of massive constituent quarks.

A comparison of the wave functions obtained in different channels is shown in fig. 3. We observe that the pion and the proton as well as the rho meson and the delta resonance have very similar wave functions. The pion and the proton, however, are significantly smaller than the rho meson and the delta resonance. We have already argued that the scalar diquark content of the nucleon and the fact that instantons produce substantial attraction in the scalar diquark channel suggest that the nucleon and pion wave functions are very similar. Analogously, one can think of the delta as consisting of a quark and a vector diquark. In this case, however, the above argument does not really apply since the d quark is not necessarily sitting in the diquark and there is no interaction that would enhance the importance of the quark-diquark part of the correlation function.

In fig. 4. we compare our results for the pion and the proton with the lattice result reported in [2, 3]. These authors have determined the Bethe Salpeter wave functions for the pion, the rho meson and the nucleon in a quenched lattice simulation. In addition to that, they have also measured wave functions after 'cooling' their configurations. This procedure relaxes any given gauge field configuration to the closest classical solution [17]. It removes essentially all gluon fluctuations except for those stabilized by topology. 'Cooling' therefore displays the contribution of instantons to the measured wave functions. Comparing the different curves shown in fig. 4. we observe that the full lattice wave functions are more compact than the instanton results while the cooled wave functions are even larger. Qualitatively it is clear why the wave functions in the instanton model extend farther out: the instanton model misses at least two effects, the Coulomb force at short distances and the string potential at large distances. Both help to localize the wave functions, although none of them seems to be of crucial importance.

This can be seen in more detail by considering the shape of the wave functions. At short distances our wave functions are quadratic in the separation, while the lattice results appear to be linear. This effect should be due to the perturbative Coulomb and spin-spin forces, since the linear behavior disappears when the wave functions are cooled. This can be checked further by applying only a few cooling sweeps (compared to the 25-50 sweeps used to extract the instanton liquid): the perturbative "quantum noise" disappears already at this level.

Finally, although the shapes are similar, the absolute size of the wave functions extracted from our model and from cooled lattice configurations do not agree. A possible reason for this behavior is the fact that there are some ambiguities related to the redefinition of the lattice scale after cooling. Even before cooling, the lattice scale depends on

the particular observable that is used to fix the units [2].

4. Conclusions

We have measured Bethe Salpeter wave functions for the pion, the proton, the rho meson and the delta resonance in the Random Instanton Liquid Model.

In all the channels considered here there is evidence that this Model leads to the formation of localized wave functions. This is based on the fact that the wave functions become stable as the longitudinal separation is increased and the observation that the wave functions measured using different sources are very similar to each other.

Furthermore, the size of these bound states are quite similar to those observed experimentally and measured on the lattice. In fact, the model reproduces most of the qualitative features of the wave functions that are observed in lattice simulations. In particular, we find that the wave functions of the pion and the proton are very similar whereas the rho meson has a significantly larger size.

The shape of the wave functions measured on the lattice are somewhat different, though. This is an important observation since the model does not include some of the physics both at very short and at very large distances, namely the perturbative and the confining forces. The size of the observed deviations is therefore a measure for the importance of these effects in the formation of hadronic bound states. After cooling the qualitative behavior of the lattice results agrees with what is observed in the Instanton Model. However, the mean square radii determined in the Instanton Model are somewhat smaller as compared to the cooled lattice reported in [3].

In conclusion we have shown that the Random Instanton Liquid Model can account

for the qualitative behavior of hadronic Bethe Salpeter wave functions. It will therefore be of great interest to extend the calculations presented in this work to include the effects of finite temperature.

5. Acknowledgements

The reported work was partially supported by the US DOE grant DE-FG-88ER40388. We acknowledge the NERSC at Lawrence Livermore where most of the computations presented in this paper were performed. We would like to thank Jac Verbaarschot for his collaboration on the Instanton Model, and very helpful discussions with John Negele on lattice results.

References

- [1] E. V. Shuryak, Rev. Mod. Phys. **65**, 1 (1993)
- [2] M.-C. Chu, M. Lissia, J. W. Negele, Nucl. Phys. **B360**, 31 (1991)
- [3] J. W. Negele, Nucl. Phys. **A553**, 47c (1993)
- [4] C. Bernard, T. A. DeGrand, C. DeTar, S. Gottlieb, A. Krasnitz, M. C. Ogilvie, R. L. Sugar, D. Toussaint, Phys. Rev. Lett. **68**, 2125 (1992)
- [5] S. Schramm, M.-C. Chu, Phys. Rev. **D48**, 2279
- [6] M. Lissia, M. C. Chu, J. W. Negele, J. M. Grandy, preprint, MIT-CTP#1922
- [7] V. Koch, E. V. Shuryak, G. E. Brown, A. D. Jackson, Phys. Rev. **D 46**, 3169 (1992)
V. Koch, preprint, SUNY-NTG-94-3

- [8] T. H. Hanson, I. Zahed, Nucl. Phys. **B374**, 227 (1992)
- [9] E. V. Shuryak, Nucl. Phys. **B203**, 116, 140, 237 (1982)
- [10] E. V. Shuryak, J. J. M. Verbaarschot, Nucl. Phys. **B** (1994), in press
- [11] E. V. Shuryak, J. J. M. Verbaarschot, Nucl. Phys. **B** (1994), in press
- [12] T. Schäfer, E. V. Shuryak, J. J. M. Verbaarschot, Nucl. Phys. **B** (1994), in press
- [13] M. C. Chu, J. M. Grandy, S. Huang, and J. W. Negele, Phys. Rev. Lett. **70**, 255 (1993); preprint, MIT-CTP#2148, hep-lat/9306002
- [14] D. I. Diakonov, V. Yu. Petrov, P. V. Pobylitsa, Phys. Lett. **B226**, 372 (1989)
- [15] E. V. Shuryak, Nucl. Phys. **B214**, 237 (1983)
- [16] D. I. Diakonov, V. Yu. Petrov, Nucl. Phys. **B245**, 259 (1984)
- [17] M. C. Chu, J. M. Grandy, S. Huang, and J. W. Negele, preprint, MIT-CTP#2269, hep-lat/9312071

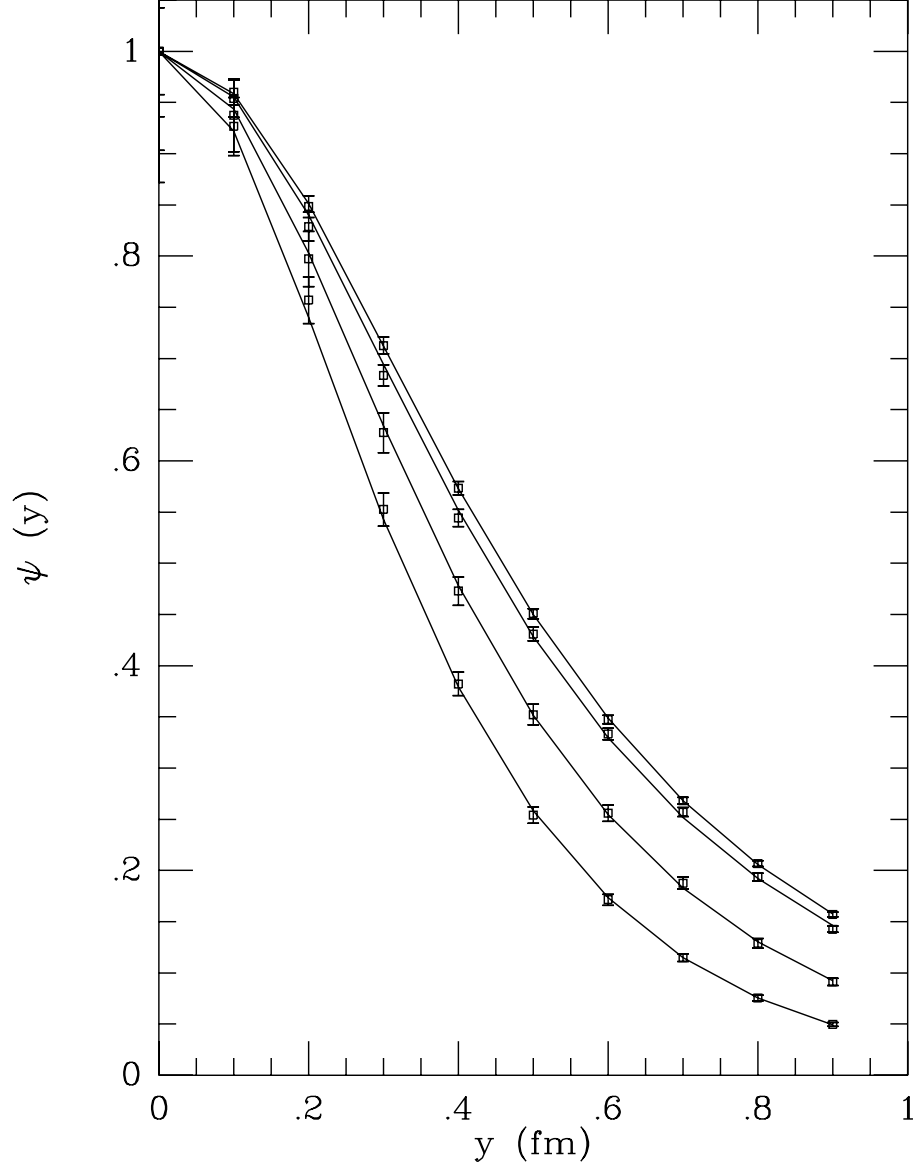


Figure 1: Pion Bethe Salpeter wave function measured at different separations $\tau = 0.5, 0.75, 1.0$ and 1.25 fm. All wave functions are normalized to one at the origin. The lowest curve corresponds to $\tau = 0.5$ fm. The solid curves show a parametrization used to extract the means square radii.

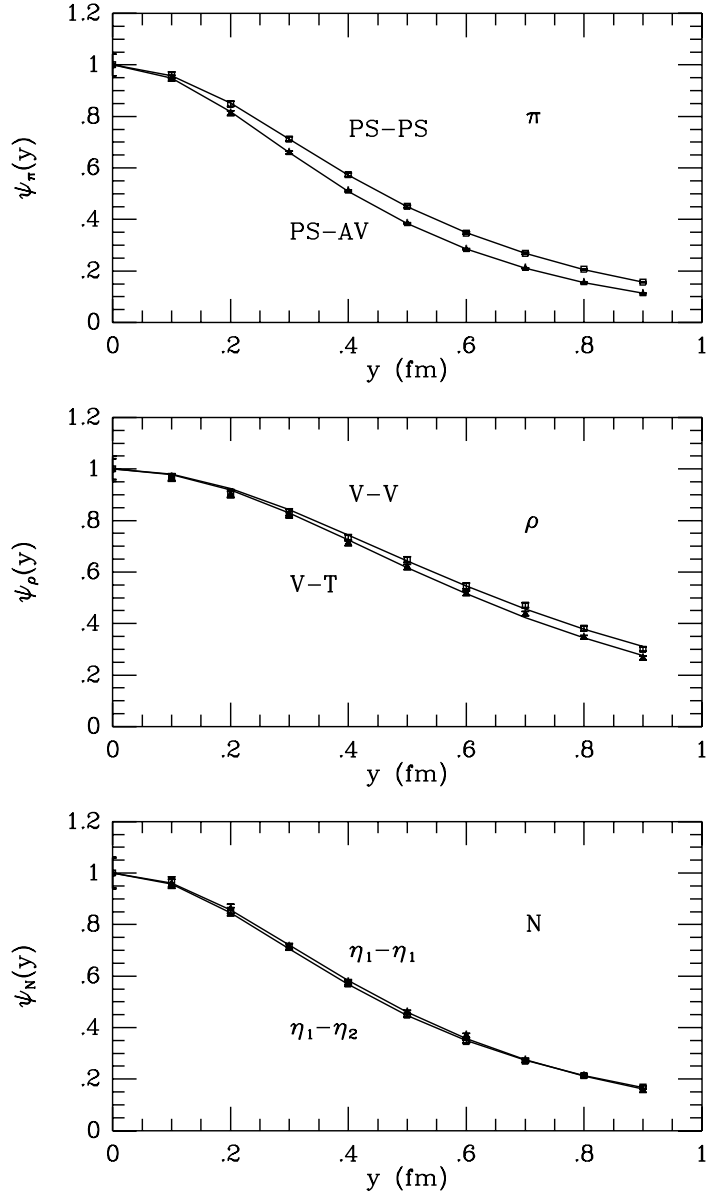


Figure 2: Bethe Salpeter wave functions for the pion, the rho meson and the nucleon as determined from diagonal and off diagonal correlation functions at $\tau = 1.25$ fm. The different correlators used are explained in the text. All wave functions are normalized to one at the origin.

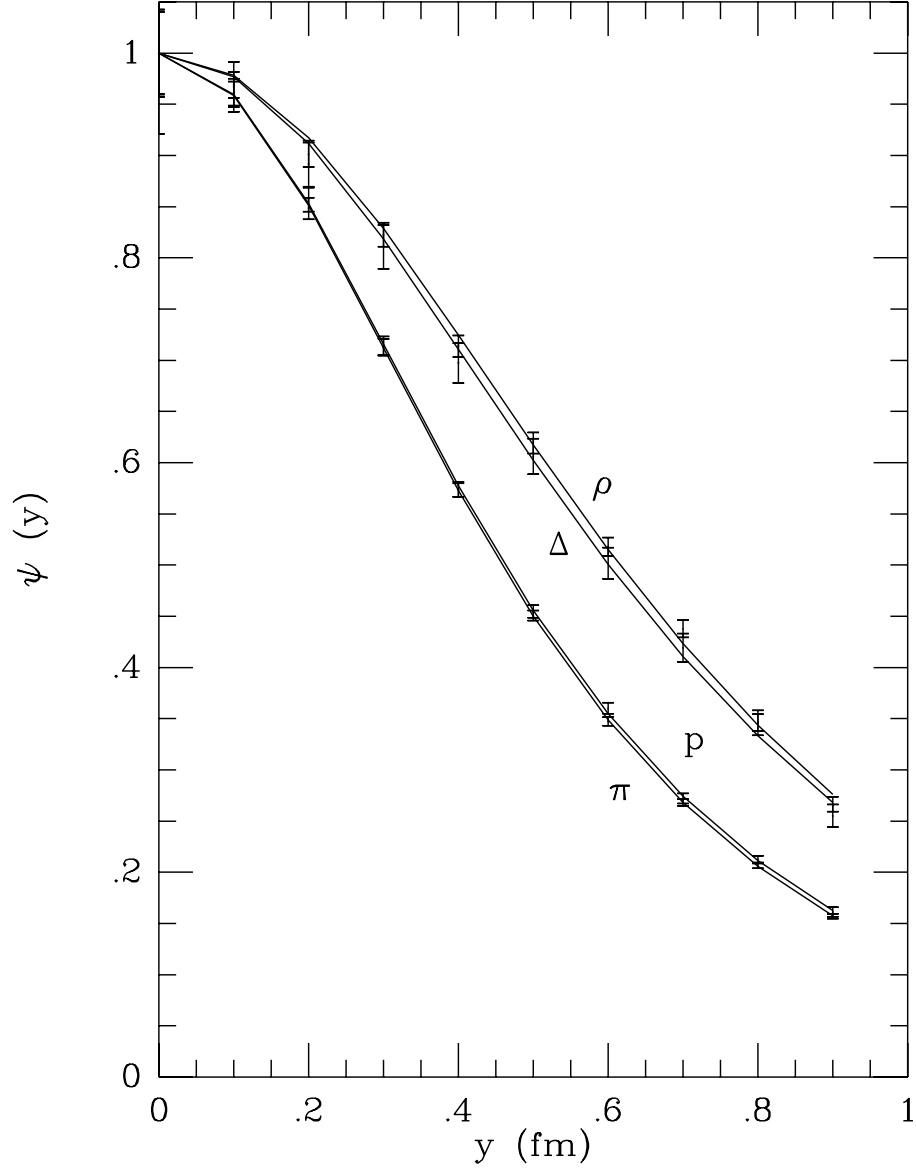


Figure 3: Bethe Salpeter amplitudes for the pion, the rho meson, the nucleon and the delta measured in the random instanton model. The wave functions are determined from diagonal correlation functions at $\tau = 1.25$ fm. They are normalized to one at the origin. The solid curves show a parametrization of the data used to extract the corresponding means square radii.

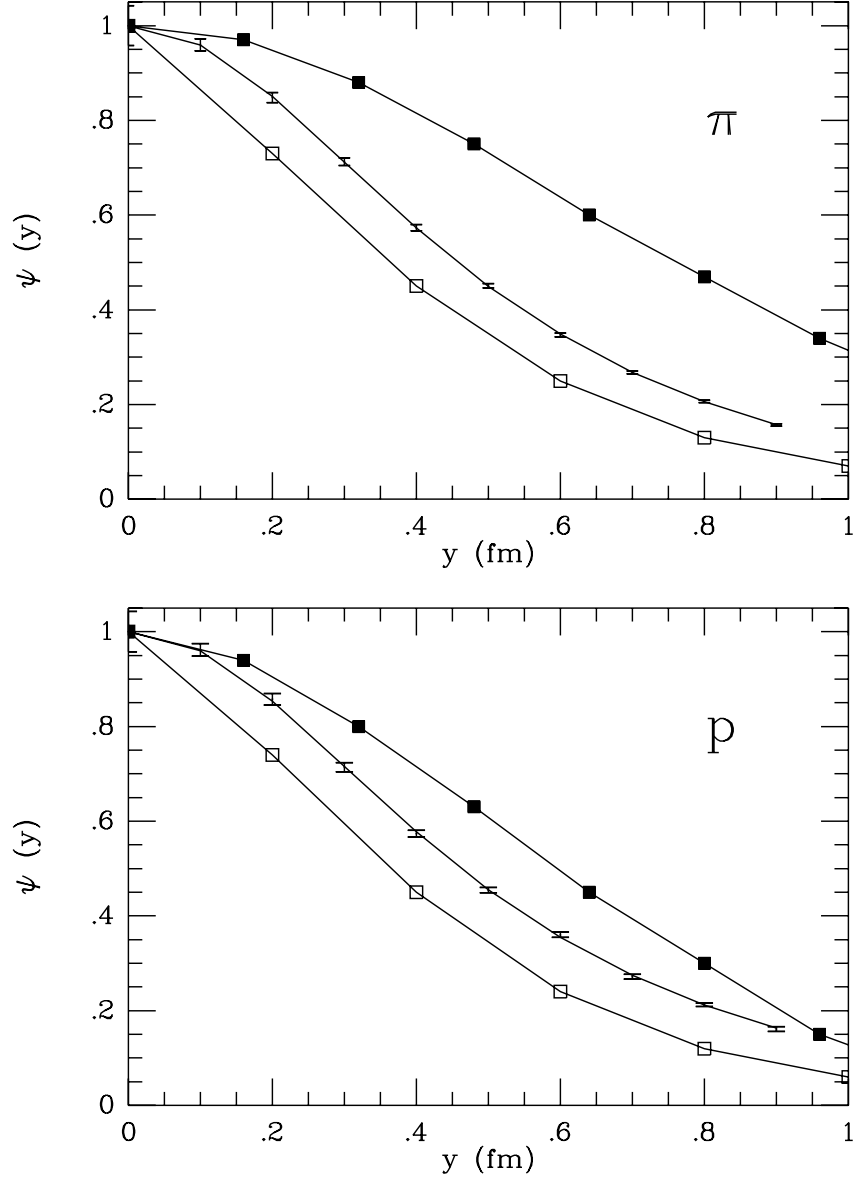


Figure 4: Comparison of pion and proton Bethe Salpeter amplitudes calculated in the random instanton model with the corresponding results of lattice calculations reported in [2,3]. The lowest curve is the full lattice result, the curve in the middle shows the result in the instanton model and the upper curve corresponds to the lattice result after cooling.